AD NUMBER
AD886372
LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; 09 AUG 1971. Other requests shall be referred to Office of Naval Research, 875 North Randolph Street, Arlington, VA 22203-1995.

AUTHORITY

ONR ltr, 13 Jan 1972

MONSANTO/WASHINGTON UNIVERSITY
ONR/ARPA ASSOCIATION

STIFFNESS OF NON-ALIGNED FIBER REINFORCED COMPOSITES

BY

R. E. LAVENGOOD AND L. A. GOETTLER

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Director of Material Sciences, Office of Naval Research.

PROGRAM MANAGER

ROLF BUCHDAHL

MONSANTO RESEARCH CORPORATION
A SUBSIDIARY OF MONSANTO COMPANY

800 N. LINDBERGH BOULEVARD   ST. LOUIS, MISSOURI 63166

D D C

AUG 9 1971

C

STIFFNESS OF NON-ALIGNED FIBER REINFORCED COMPOSITES

BY

R. E. LAVENGOOD AND L. A. GOETTLER

MAY 1971

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Monsanto Research Corp.	Unclassified
	2b. GROUP

3. REPORT TITLE

Stiffness of Non-Aligned Fiber Reinforced Composites

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

R. E. Lavengood and L. A. Goettler

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May, 1971	58	17

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. N00014-67-C-0218	
c.	HPC 71-141
d.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT This document is subject to special export controls and each transmittal to foreign governments or foreign nations may be made only with prior approval of the Director of Material Sciences, Office of Naval Research.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Office of Naval Research Washington, D. C. 20360

13. ABSTRACT

A general procedure for predicting the average Young's modulus of short fiber composites is formulated and applied to random mat, three-dimensional random prepreg, and transfer moldings. In all cases a uniform strain analysis agrees closely with experimental data for epoxy composites reinforced with graphite, stainless steel, and glass fibers. This reflects a high average stiffness in the composites.

Several limiting cases and specific prediction techniques appearing in the literature are compared for the random composites. Simple formulas are adequate for predicting the Young's modulus of these systems. The effects of component properties on the random modulus are described.

A new technique is developed for calculating the average modulus of a general axisymmetric composite by integrating over the orientation distribution.

DD FORM 1473 (PAGE 1)
1 NOV 65

S/N 0101-807-6801

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
compliance stiffness random fiber-reinforced short fiber elasticity average orientation prediction non-aligned composite material Young's modulus molding graphite fiber glass fiber stainless steel fiber epoxy						

FOREWORD

STIFFNESS OF NON-ALIGNED FIBER REINFORCED COMPOSITES

R. E. Lavengood and L. A. Goettler

Monsanto Company
800 N. Lindbergh Blvd.
St. Louis, Missouri 63166

ABSTRACT

A general procedure for predicting the average Young's modulus of short fiber composites is formulated and applied to random mat, three-dimensional random prepreg, and transfer moldings. In all cases a uniform strain analysis agrees closely with experimental data for epoxy composites reinforced with graphite, stainless steel, and glass fibers. This reflects a high average stiffness in the composites.

Several limiting cases and specific prediction techniques appearing in the literature are compared for the random composites. Simple formulas are adequate for predicting the Young's modulus of these systems. The effects of component properties on the random modulus are described.

A new technique is developed for calculating the average modulus of a general axisymmetric composite by integrating over the orientation distribution.

STIFFNESS OF NON-ALIGNED FIBER REINFORCED COMPOSITES

R. E. Lavengood and L. A. Goettler

Monsanto Company
800 L. Lindbergh Blvd.
St. Louis, Missouri 63166

Introduction

The expanding use of sheet and bulk molding compounds has increased the need for reliable techniques for predicting the stiffness of composites reinforced with short fibers. The elastic properties of specially orthotropic composites in which fibers are well aligned either parallel or perpendicular to the direction of stress can be estimated from the component properties and fiber loading by equations reported by Halpin (1). However, this is an artificial situation for short fibers, which cannot easily be put into a high degree of alignment. Instead, practical moldings usually contain a wide distribution of fiber orientation angles. The degree to which orientation can vary is illustrated by the polished longitudinal section of a flow molded part, shown in Figure 1.

The purpose of this paper is to discuss, in general, the means for integrating over the distribution of fiber angles in order to predict the Young's modulus of the molding. Nonuniform, as well as random, distributions will be considered. Comparison of various predictions with the measured moduli of epoxy composites indicates acceptable simplifying assumptions.

Similar treatments have been given to the prediction of stiffness by a laminate approximation (2,3). However, our work is more general, provides a basis for comparing various averaging techniques, and considers the treatment of three-dimensionally random and nonuniform symmetric orientation patterns in detail.

- 2 -

In this study the fibers are sufficiently long to give essentially the same stiffness as continuous reinforcement. This is an important restriction since the dependence of off-axis stiffness on the aspect ratio of short fibers is not currently known.

The loss in maximum unidirectional stiffness that occurs from randomizing the fibers in two and three dimensions to attain isotropy is illustrated in Figure 2. In addition, the random structures cannot be fabricated with as high a fiber content. Practical limits to fiber loading are indicated by the end points of the curves. The large differences which exist among the various types of composites emphasize the need for careful design.

General Treatment of the Orientation Distribution

The elastic properties of a short fiber composite can be represented by a weighted linear superposition of the stiffness or compliance contributions of the individual fibers and their associated matrix. When the orientation does not vary in the stress direction, the averaging takes place over the oriented "structural units" in the transverse cross-sectional area of the piece. This structural unit may be either microscopic or macroscopic in scale, depending on the type of composite. In the microscopic case, variations in orientation occur between adjacent fibers or neighboring groups of fibers; each of these would be considered a unit. Examples of this type of composite are random composites, and those in which the fiber reinforcement tends to form a network by extensive crossing. On the other hand, when a variation in orientation occurs across distances which are large

- 3 -

in comparison to a fiber, the structural unit is macroscopic. It may consist of a bundle of fibers or a grain composed of several bundles, all of which are aligned at the same off-axis orientation. This type of structure typically occurs in injection or transfer molding with relatively long (1/8" - 1/4") fibers.

The procedure for calculating overall Young's modulus of a composite involves the steps listed below. We take the approach of calculating the average longitudinal Young's modulus for a hypothetical uniaxial tensile stress applied in such a direction that the structure of the piece is symmetrical about the axis. By axisymmetric we mean that there are two orthogonal reflection planes intersecting the axis. Thus, the resulting structure is specially orthotropic. Similar behavior would occur in a balanced symmetric laminate under a pure tensile stress. In a more generally anisotropic structure the bending which would result from macroscopic shear coupling would greatly complicate the analysis. Although parts of this same general treatment have been followed by other investigators, we include it not only as a description of our procedure, but as a convenient outline.

1. Assumption of plane stress or plane strain.

The elasticity equations for a composite can be reduced to two dimensions by assuming that either the stress or the strain is zero in the thickness direction. In a state of plane stress, the strain in the thickness direction is not zero, even though it does not appear in the two-dimensional matrix equations. To account for the Poisson effects in this direction, the C-matrix of stiffness elements is reduced

- 4 -

to a Q-matrix as shown by Tsai (4). Similarly, for compliance the S-matrix must be changed to an R-matrix in a plane strain situation.

2. Assumption of uniformity.

It is necessary to assume that all structural elements in the composite are under either the same stress or the same strain to simplify the calculations. This can be justified by noting that uniform stress and uniform strain represent true lower and upper bounds to the elastic behavior (5). These solutions are commonly called Reuss and Voigt analyses, respectively, after the first workers to employ those assumptions.

For reasons to be discussed later, the assumption of uniform strain is found to be the more accurate and corresponds to averaging over the transverse cross-sectional area of a piece. This is in contradiction to some recently published data (17).

3. Calculation of uniaxial engineering properties.

If the reinforcing fibers have a length-to-diameter ratio well above the critical level, the engineering elastic constants can be estimated from the properties of the constituents after the manner of Halpin and Tsai (1):

$$E_{11} = E_m + v_f \left(E_f - E_m \right) \tag{1}$$

$$\nu_{12} = \nu_m + v_f \left(\nu_f - \nu_m \right) \tag{2}$$

$$E_{22} = E_m \frac{2v_f (R - 1) + (R + 2)}{v_f (1 - R) + (R + 2)} \tag{3}$$

$$G_{12} = G_m \frac{v_f (K - 1) + (K + 1)}{v_f (1 - K) + (K + 1)} \tag{4}$$

where E_m = Young's modulus of the matrix
E_f = longitudinal Young's modulus of the fibers
v_f = fiber volume fraction
ν_m = Poisson's ratio of the matrix
ν_f = Poisson's ratio of the fiber
G_m = shear modulus of the matrix
R = ratio of transverse fiber modulus to matrix modulus
K = ratio of fiber shear modulus to matrix shear modulus

In addition,

$$\nu_{21} = \frac{E_{22}}{E_{11}} \ \nu_{12} \tag{5}$$

and

$$\nu_{23} \approx 0.2 \text{ to } 0.3 \tag{6}$$

for most composites. There is no simple prediction rule for ν_{23}. In these equations X_1 is taken as the direction of fiber alignment; the $X_2 X_3$ plane is isotropic. Some improvements have been proposed for the above equations. Nielsen (6) makes an allowance for the maximum packing of fibers that becomes important at high fiber loadings. He also interprets one of the parameters as a generalized Einstein coefficient, which modifies these equations for applications to nonfibrous reinforcements. Hewitt and deMalherbe (7) propose an empirical correction to the dependence of the shear modulus on fiber volume fraction.

4. Conversion of engineering constants to uniaxial stiffness and compliance.

The stiffness (C or Q matrices) and compliance (S or R matrices) are fourth order tensors which can be transformed through angular rotations corresponding to the orientation of the structural element in the composite. The individual engineering constants cannot be used directly since they do not possess these properties. The pertinent equations for converting to stiffness and compliance are summarized in

Table I. To avoid errors due to the neglect of shear coupling terms, stiffness elements should be used if uniform strain is assumed, and compliance elements for uniform stress.

5. Rotational transformation of the elasticity elements.

Tensor theory provides transformation equations for the changes in the stiffness and compliance elements as the structural element is rotated about its x_3 axis from the in-stress direction through an angle θ to its position in the composite. The complete set of equations, appearing in (4,8), can be reduced for our purposes since the aligned composite that represents a structural element is transversely isotropic. Consequently, the number of independent elastic constants is reduced by symmetry considerations. This includes the elimination of shear coupling terms from the transformation equations. For these structures a tensile stress does not induce a macroscopic shear deformation. The transformation equations required for estimating overall Young's modulus are

$$W_{11}(\theta) = W_{11}\, m^4 + \left(2W_{12} + uW_{66}\right) m^2 n^2 \tag{7}$$
$$+ W_{22}\, n^4$$

$$W_{12}(\theta) = W_{12}\left(m^4 + n^4\right) + \left(W_{11} + W_{22} - uW_{66}\right)m^2 n^2 \tag{8}$$

$$W_{22}(\theta) = W_{11}\, n^4 + \left(2W_{12} + uW_{66}\right)m^2 n^2 + W_{22}\, m^4 \tag{9}$$

$$uW_{66}(\theta) = \left(4W_{11} - 8W_{12} + 4W_{22}\right)m^2 n^2 + \left(m^2 - n^2\right)^2 uW_{66} \tag{10}$$

where

W_{ij} represents any elastic element: C_{ij}, Q_{ij}, R_{ij} or S_{ij}

u   is a factor = 4 for C and Q
             = 1 for S and R

m = cos θ

n = sin θ

Only Equation (7) applies in three dimensions.

6. <u>Averaging over the orientation distribution.</u>

In every structural element, with orientation θ to
the stress direction,

$$\left. \begin{array}{l} \sigma_i = \displaystyle\sum_{j=1}^{6} C_{ij}(\theta)\epsilon_j \\[2em] \epsilon_i = \displaystyle\sum_{j=1}^{6} S_{ij}(\theta)\sigma_j \end{array} \right\} \qquad (11)$$

where

C_{ij} = tensorial stiffness element for the same composite
     with fibers well aligned at angle θ to the stress.

S_{ij} = corresponding tensorial compliance element.

The response of the composite is obtained by averaging the
responses of all such structural units. For uniform strain
the average stiffness is employed,

$$\bar{\sigma}_i = \sum_{j=1}^{6} \bar{C}_{ij}\,\epsilon_j \ . \qquad (12)$$

Average compliance is used with the uniform stress assumption

$$\bar{\epsilon}_i = \sum_{j=1}^{6} \bar{S}_{ij}\,\sigma_j \ . \qquad (13)$$

- 9 -

In these equations,

$$\bar{C}_{ij} = \sum_{k=1}^{N} C_{ij}(\theta_k) P_k$$

$$= {}_0\!\int^{\pi/2} C_{ij}(\theta) f(\theta) d\theta \tag{14}$$

$$= \frac{1}{V} \int\!\!\int\!\!\int_V C_{ij} \{\theta(V)\} \, dV \tag{15}$$

where k = index on the structural unit,

P_k = the fraction of the total fibers which are oriented at the angle θ_k to the applied stress,

N = the total number of different angles in the composite,

f = the probability density function of θ, i.e., the orientation distribution function,

V = the volume of the composite,

and similarly for $\bar{S}_{ij}$. Equation (14) is useful for simple distributions where the probability density function is known, e.g., a random distribution. On the other hand, when θ must be measured as a function of position in a molding, Equation (15) is more convenient.

7. <u>Calculation of average Young's modulus</u>.

The average modulus of the composite, $\bar{E}_1$, is defined by

$$\bar{\sigma}_1 = \bar{E}_1 \, \epsilon_1$$

or

$$\bar{\epsilon}_1 = \frac{1}{\bar{E}_1} \, \sigma_1 \tag{16}$$

where the 1-direction is that of the uniaxial tensile stress.

- 10 -

The pertinent equations relating $\bar{E}_1$ to the averaged elasticity elements are shown in Table II for isotropic and specially orthotropic structures. Under certain conditions no entry has been made because the corresponding relationships are too nonlinear to permit a closed form solution for $\bar{E}_1$.

All of the equations for $\bar{E}_1$ do exist for a two dimensional composite under plane stress, which is the most useful condition. In three dimensions, averaged stiffness elements other than $\bar{C}_{11}$ cannot be obtained from the simple equations given previously. The complete orientation distribution function is bivariate. Simpler but less precise procedures for calculating $\bar{E}_1$ that utilize only the $\bar{C}_{11}$ stiffness element are recommended.

Shear coupling effects can be neglected when there is proper symmetry and the same elasticity element (stiffness or compliance) is used in steps 5 and 6. After the angular transformation, but before averaging over the orientation distribution, each structural element behaves like an off-axis uniaxial composite. In addition to the shear coupling that occurs in such a generally orthotropic system, the local shear and normal stresses may also be non-zero. Any conversion of elasticity elements that neglects the non-zero shear coupling and Poisson effects would cause an error at this point.

- 11 -

We will now illustrate the application of the elasticity equations and averaging techniques to three different types of composites. These are all practical moldings made from short fiber reinforced epoxy.

Two Dimensionally Random

The isotropic "random-in-a-plane" structure results when thir composites are made from a sheet molding compound or random preforms. In this case $f(\theta)$, the probability density function which describes the orientation distribution, is a constant k, independent of θ, and equal to $2/\pi$. This reduces Equation (14) to

$$\bar{W}_{ij} = \frac{2}{\pi} \int_0^{\pi/2} W_{ij}(\theta) \, d\theta \tag{17}$$

where W represents any of the four elasticity elements.

The upper- and lower-bound predictions of the stiffness are compared in Table III. The lower bound (Reuss) estimate for Young's modulus is 35-40 percent lower than the corresponding Voigt estimate in this case. These results are obtained by integrating the appropriate elasticity elements in Equation (17) and then converting the result to the Young's modulus, $\bar{E}_1$, by use of the equations for an isotropic body presented in Table II. Estimates of the shear modulus and Poisson's ratio are also given.

With the assumption of plane stress, which is usually valid for laminates and sheet materials, the two-dimensional stiffness Q applies under the condition of constant strain.

Both the Q_{11} and Q_{12} elements must be averaged. Previous workers (9,10) have estimated $\bar{E}_1$ by equating it to $\bar{Q}_{11}$ alone. This assumes that $\left(1 - \bar{\nu}_{12}\,\bar{\nu}_{21}\right)$ is unity. Although this term is .98 for aligned composites, its neglect in the random system causes about a 10 percent error.

Substituting from Equations (7) and (8),

$$\bar{Q}_{11} = \frac{2}{\pi} \int_0^{\pi/2} Q_{11}(\theta)\,d\theta$$

$$= \frac{2}{\pi} \int_0^{\pi/2} Q_{11}\, m^4\, d\theta + \frac{2}{\pi} \int_0^{\pi/2} \left(2Q_{12} + 4Q_{66}\right) m^2 n^2\, d\theta$$

$$+ \frac{2}{\pi} \int_0^{\pi/2} Q_{22}\, n^4\, d\theta \tag{18}$$

$$\bar{Q}_{12} = \frac{2}{\pi} \int_0^{\pi/2} Q_{12}\left(m^4 + n^4\right) d\theta + \frac{2}{\pi} \int_0^{\pi/2} \left(Q_{11} + Q_{22} - 4Q_{66}\right) m^2 n^2\, d\theta$$

$$\tag{19}$$

Carrying out the integrations gives

$$\bar{Q}_{11} = \frac{3}{8} Q_{11} + \frac{1}{4}\left(Q_{12} + 2Q_{66}\right) + \frac{3}{8} Q_{22} \tag{20}$$

$$\bar{Q}_{12} = \frac{3}{4} Q_{12} + \frac{1}{8}\left(Q_{11} + Q_{22} - 4Q_{66}\right) \tag{21}$$

The true Voigt estimate for the Young's modulus is then calculated from

$$\bar{E}_1 = \frac{\left(\bar{Q}_{11} + \bar{Q}_{12}\right)\left(\bar{Q}_{11} - \bar{Q}_{12}\right)}{\bar{Q}_{11}} \tag{22}$$

giving

$$\bar{E}_1 = 8 \frac{\left(Q_{11} + 2Q_{12} + Q_{22}\right)\left(Q_{11} - 2Q_{12} + Q_{22} + 4Q_{66}\right)}{3Q_{11} + 2Q_{12} + 3Q_{22} + 4Q_{66}} \tag{23}$$

The Reuss-type lowerbound can be derived in a similar manner by summing compliances rather than stiffnesses.

$$\bar{S}_{11} = \frac{1}{\bar{E}} = \frac{2}{\pi} \int_0^{\pi/2} S_{11}(\theta)\, d\theta$$

$$= \frac{3}{8}\left(S_{11} + S_{22}\right) + \frac{1}{8}\left(2S_{12} + S_{66}\right)$$

$$= \frac{3}{8}\left(\frac{1}{E_{11}} + \frac{1}{E_{22}}\right) + \frac{1}{8}\left(\frac{-2\nu_{12}}{E_{11}} + \frac{1}{G_{12}}\right) \tag{24}$$

Both the upperbound and lowerbound predictions are functions of the ratio of fiber modulus to that of the matrix. A decrease in Young's modulus of the random composites relative to that of a longitudinally aligned system as the modular ratio increases is shown graphically in Figure 3. E_{11} increases almost in proportion to the fiber modulus in a given matrix, but since the modulus of a random structure is heavily influenced by the transverse properties, it does not increase as rapidly.

The calculations are plotted in dimensionless form as $\bar{E}_1/E_{11}$ in order to reduce the number of independent variables. The independent variables that must be specified are only four: E_f/E_m, ν_f, ν_m, and loading (when the fibers are sufficiently long to be considered infinitely long, as they are in this case). Calculations show that there is, in fact, no effect of fiber

Poisson's ratio for isotropic fibers over the range of 0.2 to 0.3. The effect of ν_m is also negligibly small. An increase in ν_m from .3 to .4 causes the small decreases in $\bar{E}_1/E_{11}$ that are noted in Table IV. There is no variation with the fiber loading in the composite. Since E_{11} does not depend on the Poisson coefficients, the reported changes in $\bar{E}_1/E_{11}$ also apply to $\bar{E}_1$ alone.

Since Equations (23) and (24) are somewhat cumbersome to use, some less precise expressions have been developed. Four of these will be compared with the rigorous Voigt and Reuss analyses in this paper. The first is a simplification of the Voigt or constant strain analysis that operates entirely with the Q_{11} element. Tsai and Pagano have shown that for most laminates the following approximations introduce very small errors (10):

$$Q_{12} = \frac{1}{4} E_{22} \qquad\qquad Q_{11} = E_{11}$$

$$Q_{66} = \frac{3}{8} E_{22} \qquad\qquad Q_{22} = E_{22}$$

and with the assumption that $\bar{E}_1 = \bar{Q}_{11}$,

$$\bar{E}_1 = \frac{3}{8} E_{11} + \frac{5}{8} E_{22} \tag{25}$$

The Reuss analysis in Equation (24) can be simplified in a manner similar to the derivation of Equation (25) by assuming $G_{12} = \frac{3}{8} E_{22}$ and $\nu_{12} = \frac{1}{3}$.

This leads to:

$$\frac{1}{\bar{E}_1}' \simeq \frac{3}{8}\left(\frac{1}{E_{11}} + \frac{1}{E_{22}}\right) + \frac{1}{8}\left(\frac{8}{3E_{22}} - \frac{2}{3E_{11}}\right)$$

$$\bar{E}_1 \simeq \frac{24 E_{11} E_{22}}{7 E_{22} + 17 E_{11}} \tag{26}$$

The equation proposed by Horio and Onogi (11) for two dimensionally random sheets,

$$\bar{E}_1 = \left(E_{11} E_{22}\right)^{1/2} \tag{27}$$

is derived from an integration of

$$E_\theta = \frac{E_{11} E_{22}}{E_{11} \sin^2\theta + E_{22} \cos^2\theta} \tag{28}$$

over the uniform random distribution. This equation is derived by a simple force balance on an off-axis material element and neglects shearing strains.

Nielsen and Chen (12) estimated the modulus of random composites by a technique similar to that applied to crystals by Huber and Schmid (13). They rotationally transformed the compliance, S_{11}, inverted to obtain $E_1(\theta)$, and then averaged the latter over the orientation distribution. Although this work is commonly regarded as a lowerbound estimate of stiffness utilizing a uniform stress analysis (2), this is not the case. The averaging is done under a uniform strain condition, but differences are introduced by the shear coupling and Poisson terms that are neglected in the equating of $\bar{E}_1(\theta)$ with the reciprocal of $\bar{S}_{11}(\theta)$. The off-axis

element in the composite is constrained, so that the local transverse and shearing stresses are nonzero.

These results are all compared with the rigorous upperbound and lowerbound predictions of the random modulus in Figures 4 and 5 for epoxy/glass and epoxy/graphite composites. The hybrid estimations of Horio-Onogi and Nielsen-Chen are seen to fall midway between the upper and lower bounds. The various analyses for random modulus all come into agreement with each other and also with E_{11} at very low volume loading, since under this condition a single phase of the composite predominates. For the loading range of interest in composites, the 3/8-5/8 rule is about 10% higher than the exact Voigt analysis, which is a true upperbound. This is to be expected because of the neglect of the Poisson factor.

The values of the parameters used for calculating the curves in Figures 4 and 5 are given in Table V. The anisotropy of the graphite fibers in the epoxy/graphite composites requires some further approximation in the use of the Halpin-Tsai equations for the unidirectional properties. Since these equations properly apply only to isotropic fibers, a different value of the fiber modulus was used for calculating E_{22} than for E_{11}. Table VI shows that changes in the transverse fiber stiffness have very little effect on the predicted random modulus. Our estimate of 4.1 x 10^6 psi for the transverse fiber modulus was obtained by back calculating from experimental data for transverse composites.

Experimental data covering the complete useful range of fiber loading for glass fiber/epoxy and graphite fiber/epoxy composites are compared with the equations in Figures 6 and 7.

- 17 -

The uniaxial engineering constants were calculated by means of Equations (1-6), using the component properties given in Table V. Fiber aspect ratio was approximately 2,000.

The glass data cluster quite tightly between the two upperbound Voigt predictions, thus justifying the use of the constant strain analysis. The lowerbound (Reuss) constant-stress analysis predicts moduli that are 35 percent low. Since the simple 3/8-5/8 rule of Equation (25) is as adequate as the more cumbersome true Voigt analysis, its use is recommended.

The graphite data are only slightly below the true upperbound Voigt prediction, again justifying the constant strain analysis. In this case the 3/8-5/8 rule of Equation (25) is about 10 percent high, probably due to neglect of the Poisson factor $\left(1 - \bar{v}_{12}\, \bar{v}_{21}\right)$. However, it may still be useful for its simplicity. The constant-stress prediction would be 75 percent low for the graphite composite which has a high fiber-to-matrix modular ratio. Comparison of the two figures has shown the graphite data to fall slightly lower than the glass data with respect to the corresponding Voigt analysis. Although this difference is not significant for our composites, it is an effect of finite fiber length. As the fiber modulus increases, larger aspect ratios are required to realize the maximum longitudinal composite stiffness.

3-D Random

A three-dimensionally random structure is frequently found in thick molded parts. In such a composite, fiber rotations may take place through two orthogonal angles. Consequently,

bivariable orientation distributions must be used to characterize the structure. This considerably complicates the averaging procedure that must be used in calculating the overall stiffness behavior. Practical stiffness estimations for 3-D random and a symmetric orthotropic structure are still possible by adopting the more approximate methods that have been previously described for the 2-D case.

The simplification that is required for treating three dimensional structures is to consider all fiber rotations as being made in the plane containing the fiber in its final position and the stress direction. Unlike the 2-dimensional case, there will now be a distribution of these planes about the stress direction throughout the composite, instead of a single common plane. By limiting our attention to the angle of rotation of each orientation element from the stress direction in its own plane, the problem is reduced to a single variable angle. However, we are, by so doing, now restricted to analyzing only those property changes that relate only to the one direction common to all the rotational planes, namely the direction of the applied stress. That is to say, a quasi two-dimensional analysis can be applied to the elements C_{11}, Q_{11}, S_{11} or R_{11}. Since the average $\bar{C}_{12}$, $\bar{C}_{22}$, etc. cannot be so determined, the exact formulae for calculating the overall Young's modulus are not useful. Instead, $\bar{E}_1$ must be estimated directly from the averaged one-one elasticity component.

Cox (14) has shown that for a three-dimensional random structure, the probability density function for the orientation angle equals $\sin(\theta)$. As in the two-dimensional case, we expect the constant strain to be preferable and so take an average of C_{11}.

$$\bar{C}_{11} = C_{11} \int_0^{\pi/2} m^4 n \, d\theta + \left(2C_{12} + 4C_{66}\right) \int_0^{\pi/2} n^3 m^2 \, d\theta + C_{22} \int_0^{\pi/2} n^5 d\theta \tag{29}$$

$$= \frac{1}{5} C_{11} + \frac{2}{15} \left(2C_{12} + 4C_{66}\right) + \frac{8}{15} C_{22}. \tag{30}$$

This can be approximately rewritten in terms of the engineering constants as

$$\bar{C}_{11} \approx \frac{1}{5} E_{11} + \frac{2}{15} \left(2\nu_{12}E_{22} + 4G_{12}\right) + \frac{8}{15} E_{22} \tag{31}$$

Assuming further that $\nu_{12} = 0.25$, $C_{66} = \frac{3}{8} E_{22}$ and $\bar{C}_{11} = \bar{E}_1$ leads to

$$\bar{E}_1 \approx \frac{1}{5} E_{11} + \frac{4}{5} E_{22}. \tag{32}$$

Figure 8 shows a comparison of this prediction with experimental stiffness data for epoxy reinforced with three dimensionally random stainless steel fibers. These fibers are isotropic and have a Young's modulus of 30×10^6 psi and Poisson's ratio of 0.3. The agreement between experiment and this simple theoretical equation is good.

A somewhat more accurate estimate of $\bar{E}_1$ can be obtained by foregoing the simplifications that lead to Equation (32). Instead, we rewrite the elasticity components on the right hand

- 20 -

side of Equation (30) in terms of the engineering constants for a transversely isotropic system, as given in Table I. The identity for $\bar{C}_{11}$ is a reduction of the first entry in that table to an isotropic system.

$$\bar{C}_{11} \equiv \frac{\left(1-\bar{v}^2\right)\bar{E}_1}{\psi_c(v)} = \left(1-v_{23}^2\right)E_{11}/5\psi_c + \frac{2}{15}\left[2\left(1+v_{23}\right)E_{12}/\psi_c + 4G_{12}\right]$$

$$+ \; 8\left(1-v_{12}\,v_{21}\right)E_{22}/15\,\psi_c \tag{33}$$

where $\quad \psi_c = \left(1+v_{23}\right)\left(1-v_{23}-2v_{12}\,v_{21}\right)$

Dividing by $\left(1-v_{23}^2\right)/\psi_c$ and taking

$$\frac{1-\bar{v}^2}{\psi_c(v)} \approx \frac{1-v_{23}^2}{\psi_c} \quad \text{gives}$$

$$\bar{E}_1 = \frac{1}{5}E_{11} + \frac{2}{15}\left[2\left(\frac{v_{12}E_{22}}{1-v_{23}}\right) + 4\left(\frac{G_{12}}{1-v_{23}^2}\right)\right] + \frac{8}{15}\left(\frac{1-v_{12}\,v_{21}}{1-v_{23}^2}\right)E_{22} \tag{34}$$

This result, while more complicated than Equation (32), gives a prediction for $\bar{E}_1$ that is closer to the true Voigt analysis. (A rigorous closed form solution, as given by Hearmon (8), can be obtained by other methods for this totally random distribution.) As an example, for a glass/epoxy composite in which the true Voigt estimate is 2.08×10^6 psi, the 1/5-4/5 rule of Equation (32) predicts a value of 2.47×10^6 psi, which is 19 percent high,

- 21 -

whereas Equation (34) is only 15 percent high at 2.39×10^6 psi. For this case, the difference is negligibly small, but the method used to derive Equation (34) will be generalized in the next section to treat the nonuniform symmetric distribution for which no simple form exists.

The true Reuss estimate for the lowerbound can be easily calculated for both isotropic and orthotropic structures because the Young's modulus $\bar{E}_1$ is the reciprocal of $\bar{S}_{11}$. This can be calculated by carrying out the quasi 2-D analysis as was done for $\bar{C}_{11}$ in Equation (29). This integration gives

$$\bar{E}_1 = \frac{1}{\bar{S}_{11}} = \left[\frac{1}{5} S_{11} + \frac{2}{15} (2S_{12} + S_{66}) + \frac{8}{15} S_{22} \right]^{-1} \tag{35}$$

As with the 2-D lower bound this expression can be simplified by assuming $G_{12} = 3/8\ E_{22}$ and $v_{12} = 1/3$. This yields

$$\bar{E}_1 \approx \frac{9\ E_{11}E_{22}}{8\ E_{11} + E_{22}} \tag{36}$$

In the case of the example epoxy/glass composite mentioned previously, the true lowerbound estimate calculated from Equation (35) is 1.53×10^{-6} psi, which is 26 percent below the corresponding Voigt analysis.

To put the results for random systems into perspective, the reader is refered back to Figure 2 where plots of the successful 3/8-5/8 rule for 2-D composites and the 1/5-4/5 rule for 3-D composites are compared with uniaxial longitudinal and transverse Young's moduli for glass/epoxy composites.

Symmetric Nonuniform Distributions

When a part is fabricated from bulk molding compound, the structure is neither well aligned nor random. A tendency toward good alignment can be obtained by tailoring the flow geometry of the part, but significant angular deviations always exist among the fibers, so that the uniaxial equations cannot be accurately applied. On the other hand, unless the fibers are extremely short ($l/d < 20$), enough flow orientation will occur to invalidate any assumption of randomness.

If the molding has axial symmetry, and the gate location is also symmetrical about the axis, the resulting orientation pattern will be symmetrical, although macroscopically nonuniform. If the end gate is small, the structure of the molding comprises a core of transversely oriented fibers surrounded by an envelope of orientation parallel to the axis (15). A typical orientation distribution measured in a 1/4" x 1" x 6" bar appears in Figure 9. It is possible to measure and characterize such a distribution of fiber angles in terms of a mean direction and a standard deviation about that direction as a function of the coordinate position in the molding (16). Since this is a difficult and time consuming task, it is not recommended as a routine analytical procedure. However, advances are being made in relating the expected orientation distribution to the mold geometry and molding variables. This will eventually allow the complete a priori prediction of stiffness at the design level.

- 23 -

The fiber orientation in any plane cut through the molding frequently follows a normal distribution for which the mean and standard deviation are functions only of position in the transverse cross-section (see Fig. 10).

$$\phi_i = \bar{\phi}_1 \left(x_2, x_3 \right)$$

$$s_{\phi_i} = s_{\phi_i} \left(x_2, x_3 \right)$$

$$i = 2,3$$

(37)

The azimuthal angle about the axis, $\bar{\theta}_1 \left(x_2, x_3 \right)$, can be calculated from a knowledge of the ϕ-distributions in two orthogonal planes under the assumption that these two distributions are independent. This is, in practice, an acceptable assumption for the x_2 and x_3 planes, although it is not rigorously valid. The local average θ_1 at any point in the composite is then given by

$$\bar{\theta}_1 \left(x_2, x_3 \right) = -\int_{-\infty}^{\infty} -\int_{-\infty}^{\infty} \theta_1 \left(\phi_2, \phi_3 \right) N \left[\phi_2, \bar{\phi}_2 \left(x_2, x_3 \right), s_{\phi_2} \left(x_2, x_3 \right) \right] \cdot$$

$$N \left[\phi_3, \bar{\phi}_3 \left(x_2, x_3 \right), s_{\phi_3} \left(x_2, x_3 \right) \right] d\phi_2 \, d\phi_3 \qquad (38)$$

where N is the normal distribution of ϕ_1 with mean $\bar{\phi}_1$ and standard deviation s_{ϕ_1} and

$\theta_1 \left(\phi_2, \phi_3 \right)$ is a geometrical relationship derived from the equations for spherical coordinate systems, given by

$$\theta_1 = \tan^{-1} \left(\sqrt{\operatorname{ctn}^2 \phi_2 + \tan^2 \phi_3} \right) \qquad (39)$$

- 24 -

In a similar manner an elasticity element can also be averaged over the local distribution as follows:

$$\bar{W}_{ij}(x_2, x_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{ij}\left\{\theta_1(\phi_2, \phi_3)\right\} N(\phi_2) N(\phi_3) \, d\phi_2 \, d\phi_3 \quad (40)$$

The local values of either the polar angle or the stiffness element must next be averaged over the cross-sectional area of the specimen in order to relate to the overall elastic behavior. The resulting overall averages are designated with a double bar. The integrations are best performed numerically using a computer.

Assuming constant strain, the C_{11} element is averaged over both the microscopic, or local, and macroscopic variations in the orientation angle after first dividing by the factor $\left(1 - v_{23}^2\right)/\psi_c$, as was done to arrive at Equation (33) previously.

$$\bar{\bar{E}}_1 = E_{11} \, F\left[\cos^4 \theta_1\right] + 2\left[\frac{E_{12}}{1 - v_{23}} + \frac{2G_{12}\,\psi_c}{1 - v_{23}^2}\right] F\left[\sin^2 \theta_1 \cos^2 \theta_1\right]$$

$$+ \frac{1 - v_{12}\,v_{21}}{1 - v_{23}^2} E_{22} \, F\left[\sin^4 \theta_1\right] \quad (41)$$

where $F\left[g\left\{\theta_1\right\}\right]$ is the operator $\frac{1}{A} \int_A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g\left\{\theta_1(\phi_2, \phi_3)\right\}$

$$N(\phi_2) \, N(\phi_3) \, d\phi_2 \, d\phi_3 \, dA$$

Figure 11 shows a comparison of this theoretical stiffness
analysis based on measured orientation distributions with the
measured Young's moduli for bars transfer molded from 40 to 50
v/o of 1/8" E-glass fibers in an epoxy matrix. Although some
fiber breakage occurred during molding, the average aspect ratio
in the molded bars was still 200. For a fiber-to-matrix modular
ratio of only 25, these fibers are sufficiently long to contribute
the same stiffness as a continuous filament. The predicted
moduli are about 13 percent above the measured values for the
entire range of distributions tested. This discrepancy appears
to arise from the approximate treatment given to the factors of
Poisson coefficients. Equation (41) implies that the quantity

$$\left(1 - \nu_{23}\,\nu_{32}\right)\Big/\left(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{12}\nu_{23}\nu_{31}\right)$$

is invariant under the averaging operator $F\left[g\left\{\theta_1\right\}\right]$,
when there is no a priori reason to expect such constancy.
However, the utility of the method is greatly enhanced by the
observation that, for the random geometry previously treated,
it also predicts a Young's modulus about 15 percent higher than
the true Voigt analysis. Consequently,

    i)   a very accurate value for the modulus, within a couple
            percent, can be obtained by taking 87 percent of the
            result obtained by this approximate treatment.

    ii)   the stiffness data obtained on an axisymmetric sample
            can be closely described by a Voigt-type analysis that
            assumes constant strain.

Discussion

The experimental measurements of Young's modulus show that for both random and axisymmetric composites the uniform strain analysis is accurate to within 15 percent. There are two structural characteristics of these samples that are requisites for constant-strain averaging. First, the overall structure of the composite must be specially orthotropic with respect to the load direction. This prevents the development of macroscopic shear strains under a tensile load. All structural elements in a transverse plane will then be under the same tensile strain. Local shearing strains, which might exist around the individual structural elements, will be of small scale and will tend to cancel.

Secondly, the averaging must be done only over a transverse plane. If major variations in structure occur both across and along the composite, a double averaging procedure becomes necessary. The equilibrium of forces requires that all normal sections along the axis be under a uniform load which, for constant cross-section, reduces to a uniform stress rather than uniform strain. The integral in Equation (15) can be expanded to

$$\iiint_V dV \qquad \int_\ell \iint_A dA_\varepsilon \text{ constant } d\ell_\sigma \text{ constant} \qquad (42)$$

where the order of integration is not specified. After averaging C_{11}, for example, at constant strain over cross-sectional area A, the compliance, S_{11}, can be calculated or estimated and then integrated (at constant stress) over the length dimension, ℓ. But this result is not unique; reversing

- 27 -

the order of integration will change the calculated average stiffness. When the orientation changes occur in one direction only, Equation (42) correctly reduces to the proper Voigt or Reuss average.

It is clear for the transfer moldings, where no axial variation in orientation occurs, that the averaging should be carried out in a transverse plane under the uniform strain condition. This is the Voigt type of analysis. However, the random composites show an equal distribution in orientation of the structural elements both along and perpendicular to the stress axis. Depending on the order of integration, application of Equation (42) results in either the ordinary Voigt or Reuss predictions, since the integrand in the second integration would always be a constant. Unlike the case for transfer moldings, it is not possible to predetermine the correct assumption of uniformity. However, our data on random composites show that, to a very close approximation, the Voigt uniform strain analysis applies to random systems, whereas, depending on the fiber-to-matrix modular ratio, the lowerbound Reuss analysis may be more than 75 percent low.

Summary and Conclusions

Practical moldings of short fiber composites do not have a high degree of fiber alignment. Instead, the fiber orientation is nonuniform on either a microscopic or a macroscopic scale in relation to the mold dimensions. Calculation of the average Young's modulus requires an integration of the elastic constants across the distribution of fiber orientation angles. A general procedure is formulated and applied to three types of composites. In order to avoid shear effects the structure must either be a balanced symmetric laminate or the orientation pattern must be axisymmetric to the applied tensile stress. Errors from neglected shear coupling and Poisson terms will arise if the steps in the averaging procedure are not performed in the proper sequence.

The effects of fiber loading, fiber-to-matrix modular ratio, and component Poisson coefficients on the Young's modulus of planar random composites are computed under the assumptions of uniform stress or uniform strain. The stiffness relative to an aligned composite decreases as the modular ratio increases and the Poisson coefficients have no significant effect in either case. In comparison with other theories, the uniform strain (Voigt) analysis gives the best agreement with experimental data on epoxy/glass and epoxy/graphite composites. Similar equations are derived for three dimensionally random composites.

- 29 -

A new technique is developed for analyzing stiffness in nonuniform axisymmetric composites as would be produced by injection or transfer molding. It can predict the Young's modulus from measured fiber orientation distributions within five percent.

In all cases the measured average modulus lies close to the predicted upperbound, which is based on an assumption of uniform strain in the stiffness analyses.

References

1. Halpin, J. C., "Stiffness and Expansion Estimates for Oriented Short Fiber Composites," J. Composite Materials, 3, 732 (1969).

2. Halpin, J. C. and N. J. Pagano, "The Laminate Approximation for Randomly Oriented Fibrous Composites," J. Composite Materials 3, 720 (1969).

3. Halpin, J. C., K. Jerine, and J. M. Whitney, "The Laminate Analogy for 2 and 3 Dimensional Composite Materials", J. Composite Materials, 5, 36 (1971).

4. Tsai, S. W., "Mechanics of Composite Materials," AFML-TR-66-149, Part II, (1966).

5. Paul, B., "Prediction of Elastic Constants of Multiphase Materials," Trans A.I.M.E., 218, 36 (1960).

6. Nielsen, L. E., "Generalized Equation for Elastic Moduli," J. Appl. Physics, 41, (Oct. 1970).

7. Hewitt, R. L. and M. C. deMalherbe, "An Approximation for the Longitudinal Shear Modulus of Continuous Fiber Composites," J. Composite Materials, 4, 280 (1970).

8. Hearmon, R. F. S., "An Introduction to Applied Anisotropic Elasticity," Oxford University Press, (1961).

9. Werren, F. and C. B. Norris, "Mechanical Properties of a Laminate Designed to be Isotropic," Forest Products Laboratory Report, No. 1841, (May 1953).

10. Tsai, S. W. and N. J. Pagano, "Invariant Properties of Composite Materials," Composite Materials Workshop, ed. by Tsai, Halpin, and Pagano, Technomic Publishing Co., Stanford, Conn. p. 233 (1968).

11. Horio, M. and S. Onogi, "Dynamic Measurements of Physical Properties of Pulp and Paper by Audiofrequency Sound," _J. Appl. Phys._, 22, 971 (1951).

12. Nielsen, L. E. and P. E. Chen, "Young's Modulus of Composites Filled with Randomly Oriented Fibers," _J. Materials_, 3 (2) 352 (1968).

13. Huber, A. and E. Schmid, _Helv. Phys. Acta_, 7, 620 (1934).

14. Cox, H. L., "The Elasticity and Strength of Paper and Other Fibrous Materials," _Br. J. Appl. Phys._, 3, 72 (1952).

15. Goettler, L. A., "Controlling Flow Orientation in Molding of Short-Fiber Compounds," _Modern Plastics_, 140 (April 1970).

16. Goettler, L. A., "Flow Orientation of Short Fibers in Transfer Molding," 25th S.P.I. Reinforced Plastics/Composites Div. Meeting, Section 14-A, (Feb. 1970).

17. Brody, H. and I. M. Ward, "Modulus of Short Carbon and Glass Fiber Reinforced Composites," _Polymer Eng. and Sci._, 11, 139 (1971).

List of Tables

Table I. Equations for Calculation of the Uniaxial Aligned Elasticity Elements from the Engineering Constants

A. Uniform strain; plane strain or 3-D

$$C_{11} = \left(1 - v_{23}^2\right) E_{11}/\psi_c$$

$$C_{22} = \left(1 - v_{12} v_{21}\right) E_{22}/\psi_c$$

$$C_{12} = v_{21} \left(1 + v_{23}\right) E_{11}/\psi_c$$

$$C_{66} = G_{12}$$

$$\psi_c = \left(1 + v_{23}\right)\left(1 - v_{23} - 2 v_{12} v_{21}\right)$$

B. Uniform strain; plane stress

$$Q_{11} = E_{11}/\psi_Q$$

$$Q_{22} = E_{22}/\psi_Q$$

$$Q_{12} = v_{21} E_{11}/\psi_Q$$

$$Q_{66} = G_{12}$$

$$\psi_Q = \left(1 - v_{12} v_{21}\right)$$

C. Uniform stress; plane strain

$$R_{11} = \left(1 - v_{12} v_{21}\right)/E_{11}$$

$$R_{22} = \left(1 - v_{23}^2\right)/E_{22}$$

$$R_{12} = - v_{12} \left(1 + v_{23}\right)/E_{11}$$

$$R_{66} = 1/G_{12}$$

D. Uniform stress; plane stress or 3-D

$$S_{11} = 1/E_{11}$$

$$S_{22} = 1/E_{22}$$

$$S_{12} = - v_{12}/E_{11}$$

$$S_{66} = 1/G_{12}$$

Table II

Formulas for the Average Young's Modulus
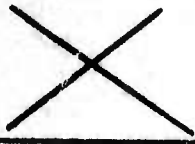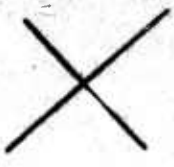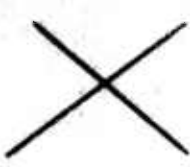
	Isotropic		Orthotropic	
	Uniform Stress	Uniform Strain	Uniform Stress	Uniform Strain
3-D	$\bar{\bar{E}}_1 = \dfrac{1}{\bar{S}_{11}}$	$\bar{\bar{E}}_1 = \dfrac{\left(\bar{C}_{11} - \bar{C}_{12}\right)\left(\bar{C}_{11} + 2\bar{C}_{12}\right)}{\left(\bar{C}_{11} + \bar{C}_{12}\right)}$	$\bar{\bar{E}}_1 = \dfrac{1}{\bar{S}_{11}}$	$\times$
Plane stress	$\bar{\bar{E}}_1 = \dfrac{1}{\bar{S}_{11}}$	$\bar{\bar{E}}_1 = \dfrac{\left(\bar{Q}_{11} + \bar{Q}_{12}\right)\left(\bar{Q}_{11} - \bar{Q}_{12}\right)}{\bar{Q}_{11}}$	$\bar{\bar{E}}_1 = \dfrac{1}{\bar{S}_{11}}$	$\bar{\bar{E}}_1 = \bar{Q}_{11} - \dfrac{\left(\bar{Q}_{12}\right)^2}{\bar{Q}_{22}}$
Plane strain	$\bar{\bar{E}}_1 = \dfrac{\bar{R}_{11} - 2\bar{R}_{12}}{\left(\bar{R}_{11} - \bar{R}_{12}\right)^2}$	$\bar{\bar{E}}_1 = \dfrac{\left(\bar{C}_{11} - \bar{C}_{12}\right)\left(\bar{C}_{11} + 2\bar{C}_{12}\right)}{\left(\bar{C}_{11} + \bar{C}_{12}\right)}$	$\times$	$\times$

Table III

Rigorous Predictions for the Stiffness of a 2-D Random Composite

$$E_{11} = 5.29 \times 10^6 \text{ psi}$$

$$E_{22} = 1.76 \times 10^6 \text{ psi}$$

$$G_{12} = .41 \times 10^6 \text{ psi}$$

$$v/o = 50$$

$$\nu_{12} = .3$$

$$\nu_{21} = .1$$

$$\nu_{23} = \nu_{32} = .2$$

Assumptions		Elasticity Element Used	$\bar{E}_1 \times 10^{-6}$ psi	$\bar{G} \times 10^{-6}$ psi	ν
Uniform	Plane				
Strain (Voigt)	Strain	C	2.52	.98	.28
Strain	Stress	Q	2.66	.98	.36
Stress (Reuss)	Strain	R	1.60	.61	.32
Stress	Stress	S	1.74	.60	.44

Table IV

Effect of Matrix Poisson's Ratio on the Stiffness of a 2-D Random Composite

Percent decrease in $\bar{E}_1/E_{11}$ caused by an increase in the Poisson's ratio of the matrix, ν_m from .3 to .4

E_f/E_m	20	200
Voigt	1.0	0.3
Reuss	2.5	3.6

Table V

Component Material Properties for Epoxy/Glass and Epoxy/Graphite
Composites

Parameters	Material	
	Epoxy/E-Glass	Epoxy/Hercules HT Graphite
$E_f \times 10^{-6}$, psi	10.5	32.*
$E_m \times 10^{-6}$, psi	.41	.41
ν_f	.22	.22
ν_m	.35	.35
$G_f \times 10^{-6}$	4.3	4.0
$G_m \times 10^{-6}$	.15	.15

*4.1×10^6 psi used for predicting E_{22} for the composite.

Table VI

Effect of Changes in the Transverse Modulus of Graphite Fiber
on the Stiffness of a 2-D Random Epoxy Composite

Transverse fiber modulus $\times 10^{-6}$, psi	Transverse fiber modulus relative to matrix modulus	Composite stiffness $\bar{E}_1/E_{11}$ at 40 v/o loading by Voigt analysis
1.	2.4	.369
4.1	10.0	.377
5.	12.2	.378
10.	24.4	.381

List of Figures

Figure 1.   Polished Longitudinal Section of a Flow Molded Part.
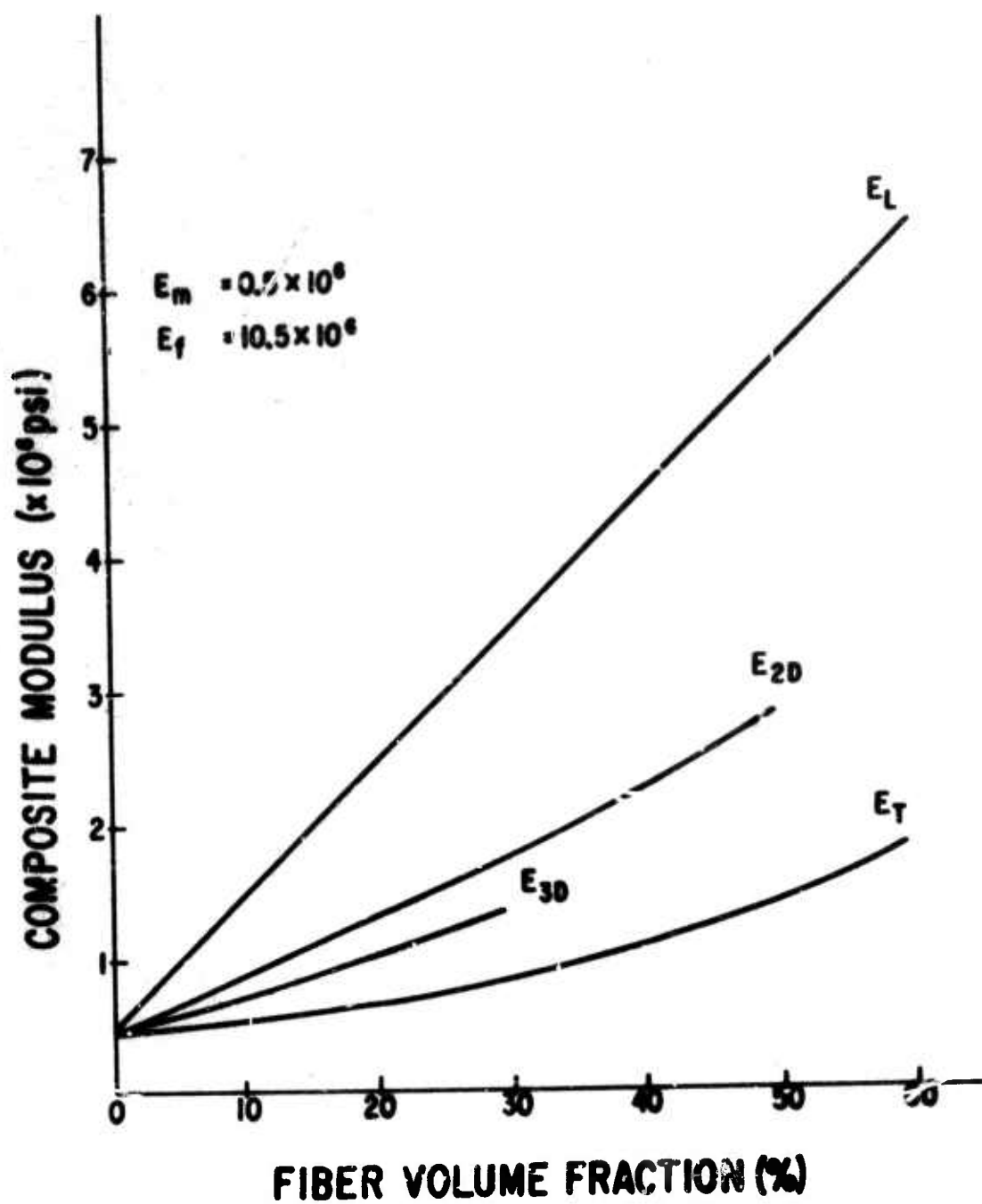
Figure 2. Comparative Stiffness of Various Types of Glass Fiber/Epoxy Composites.
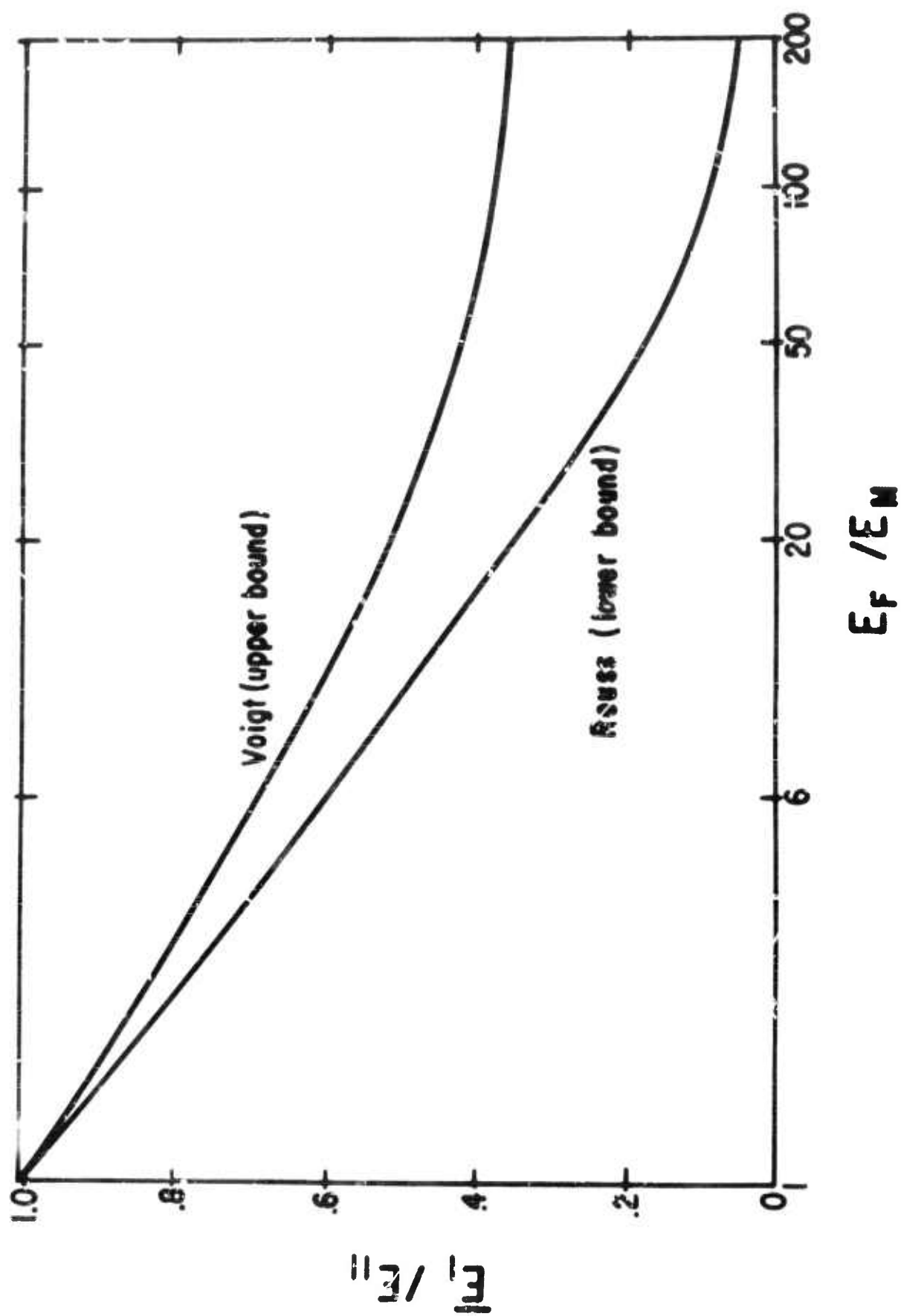
Figure 3. Bounds on the Elastic Modulus of 2-D Random Mat Containing 40 Volume Percent Fiber.
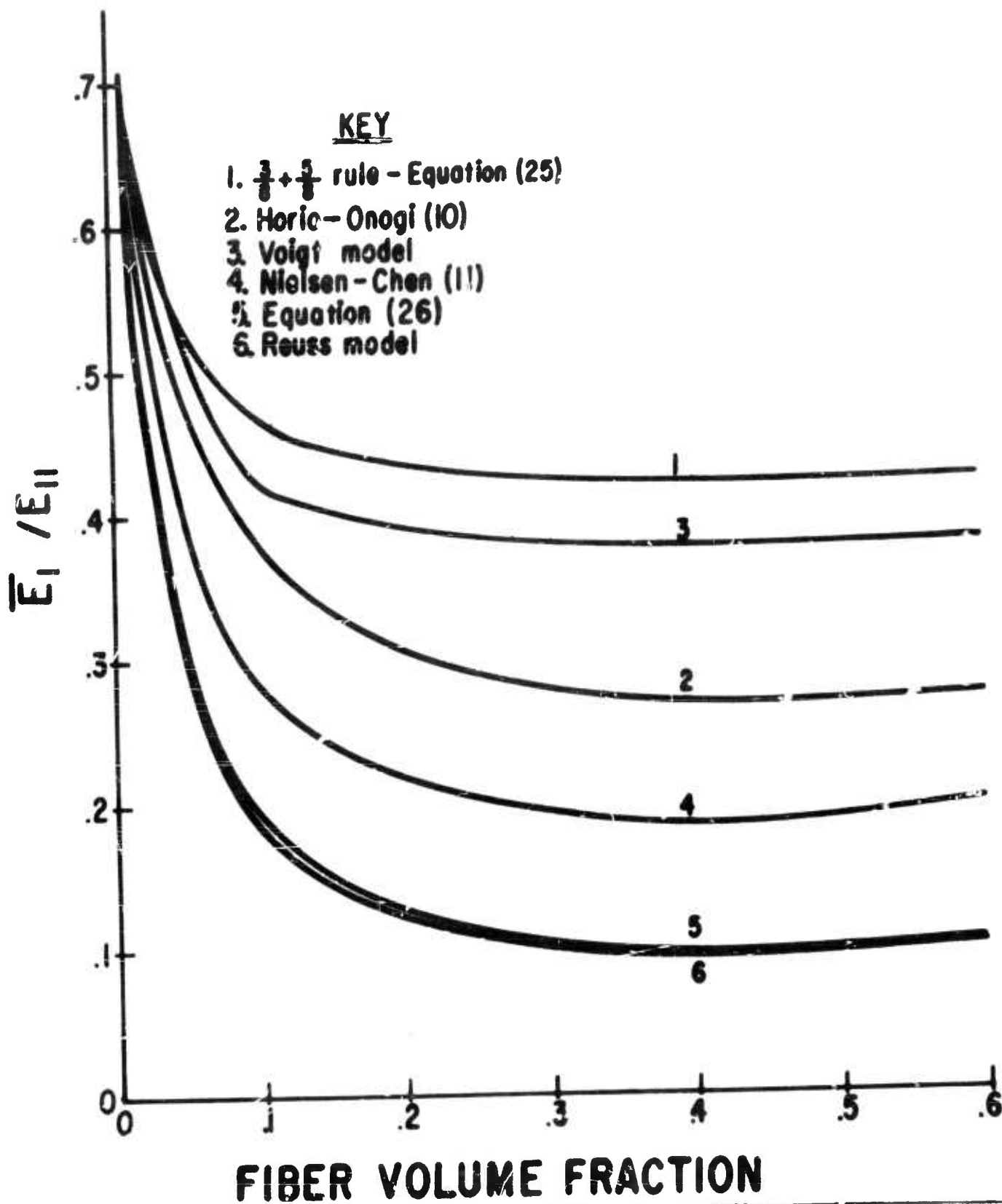
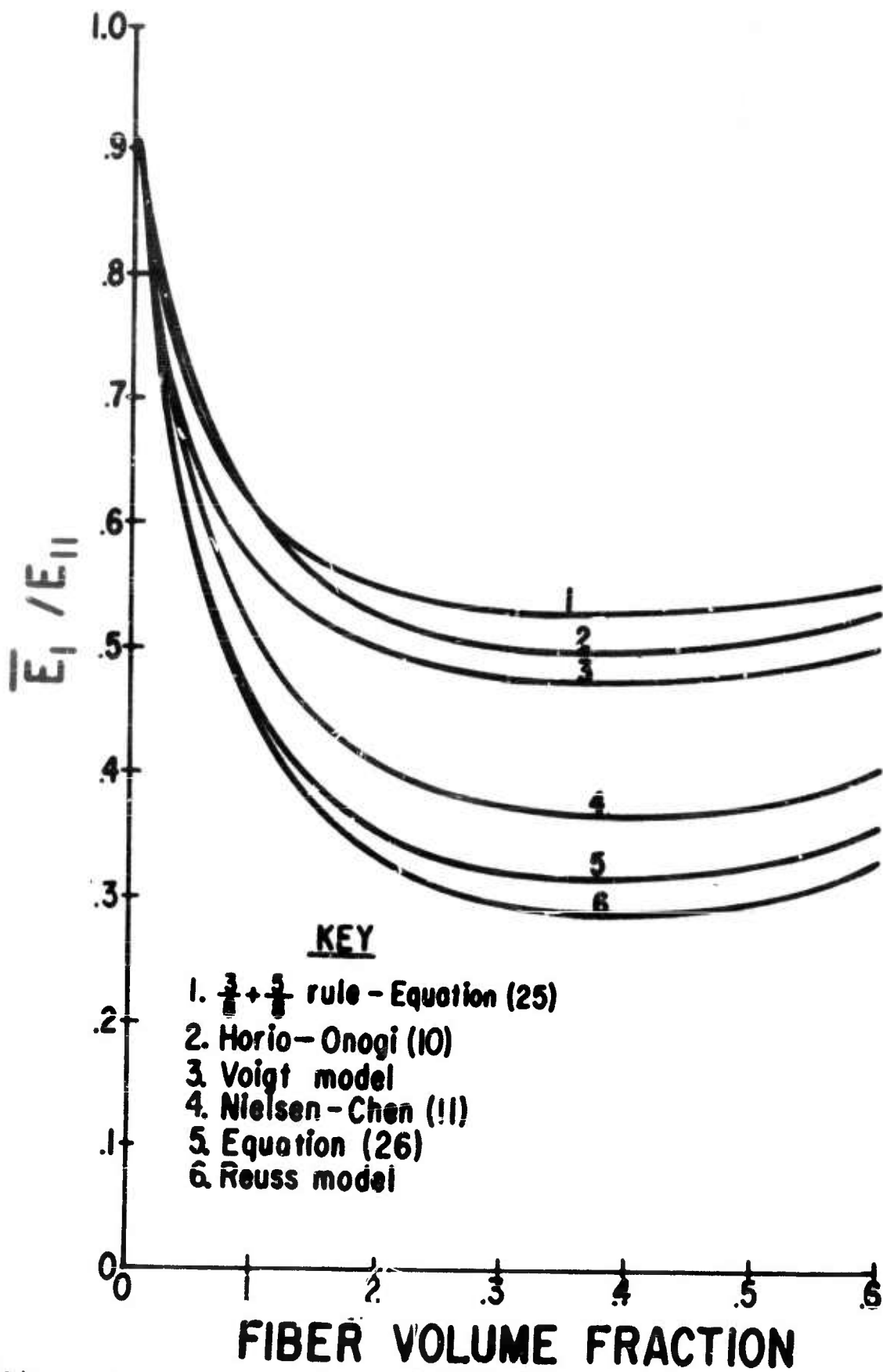Figure 4. Young's Modulus Predictions in 2-D Random Glass Fiber/Epoxy Composites.

Figure 5.   Young's Modulus Predictions in 2-D Random Graphite
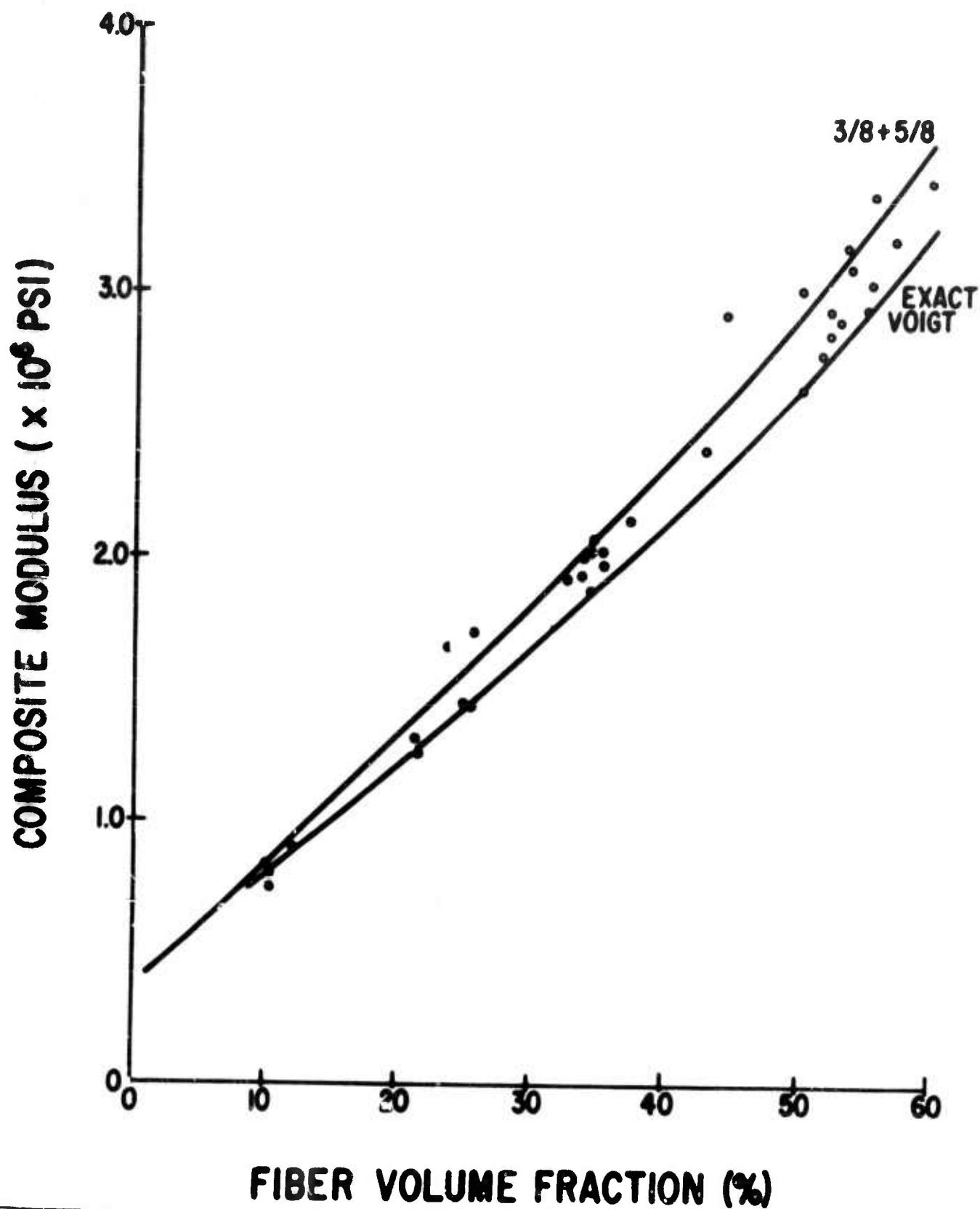Fiber/Epoxy Composites.

Figure 6.   Young's Modulus of a Planar Composite of Random Glass Fiber in an Epoxy Matrix.
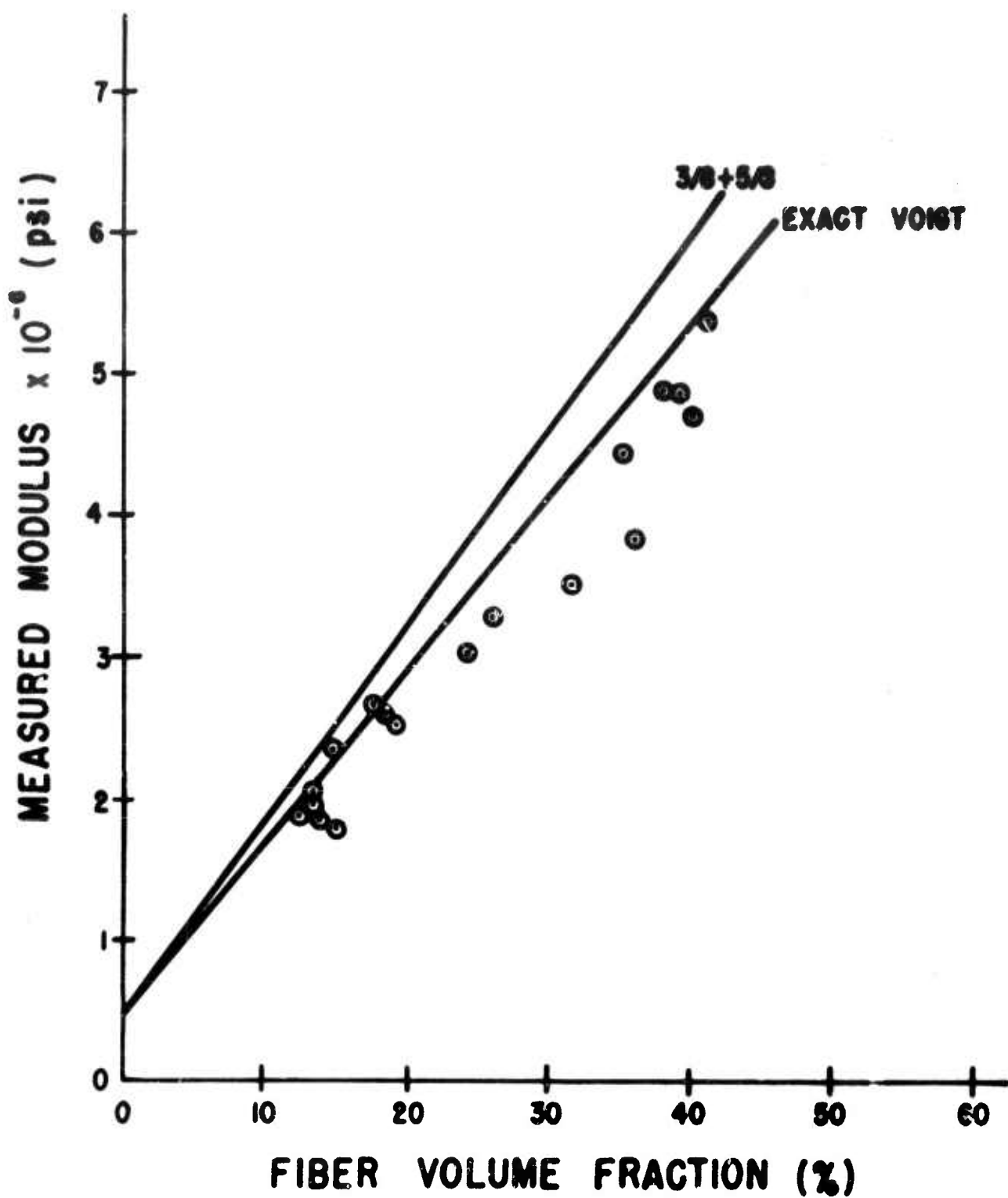
Figure 7.   Young's Modulus of a Planar Composite of Random Graphite
Fiber in an Epoxy Matrix.
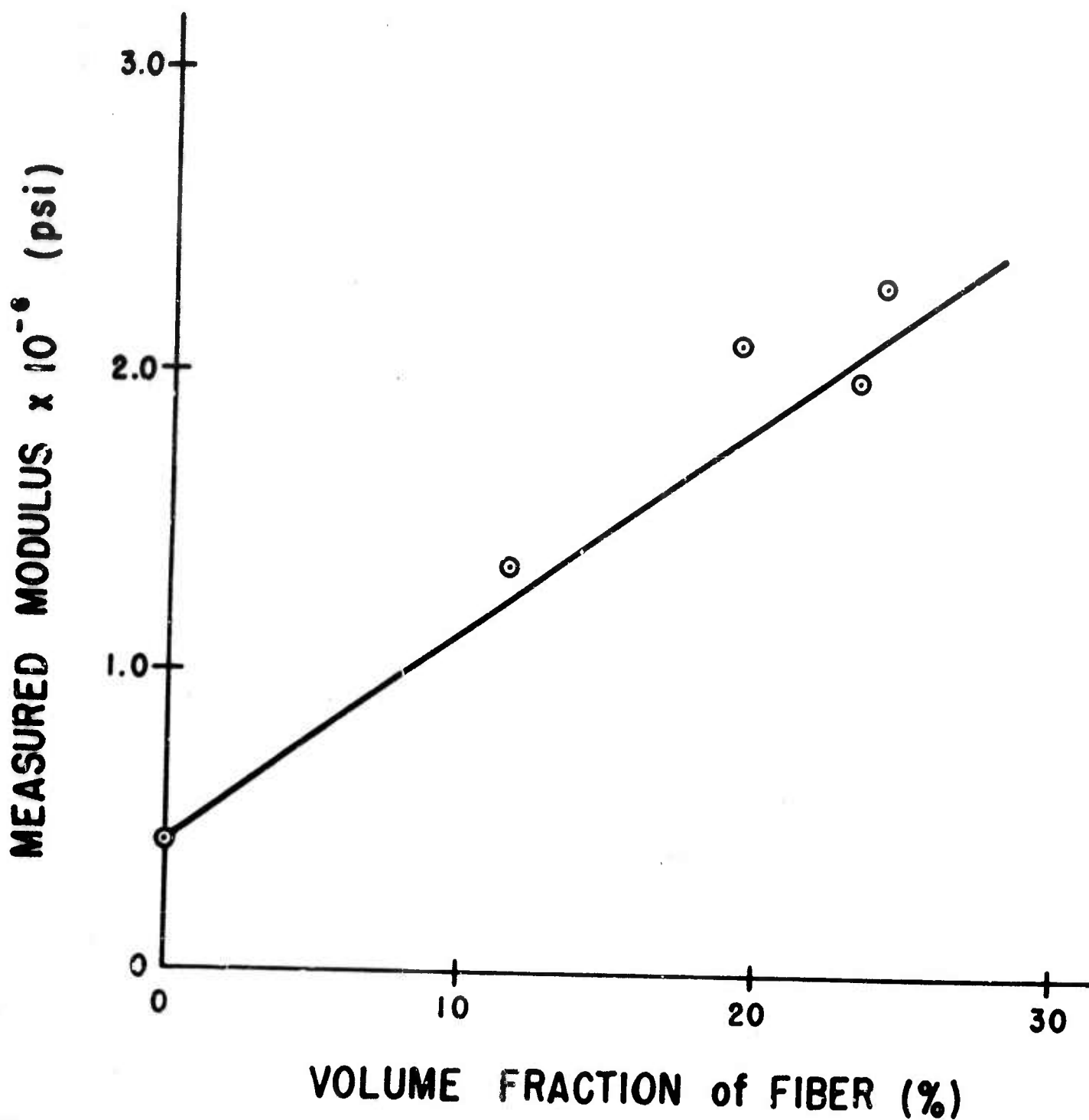
Figure 8. Young's Modulus for a Three-Dimensionally Random Composite of Stainless Steel Fiber in an Epoxy Matrix.
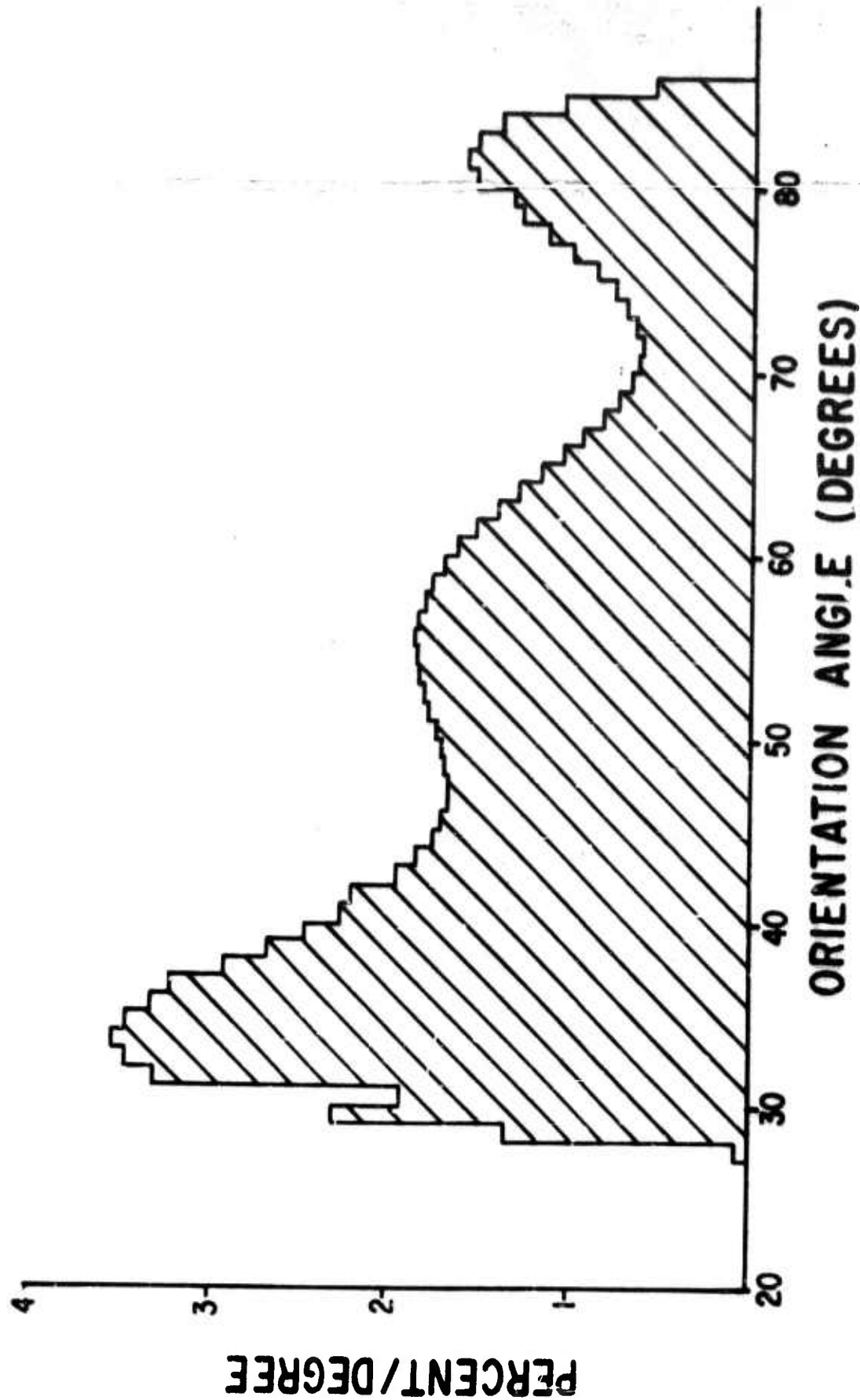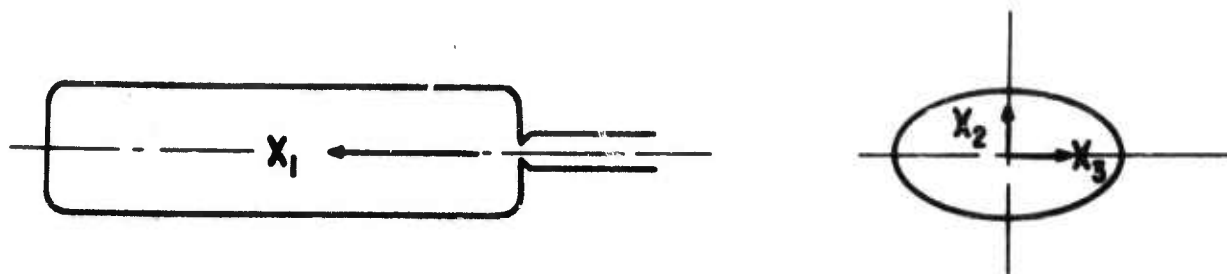
Figure 9. Orientation Distribution in a 1/4" x 1" x 6" Transfer
Molded Bar; 49 v/o 1/8" E-Glass in Epoxy, ℓ/d = 200.

X_1 = flow direction during molding

= stress direction during tensile testing

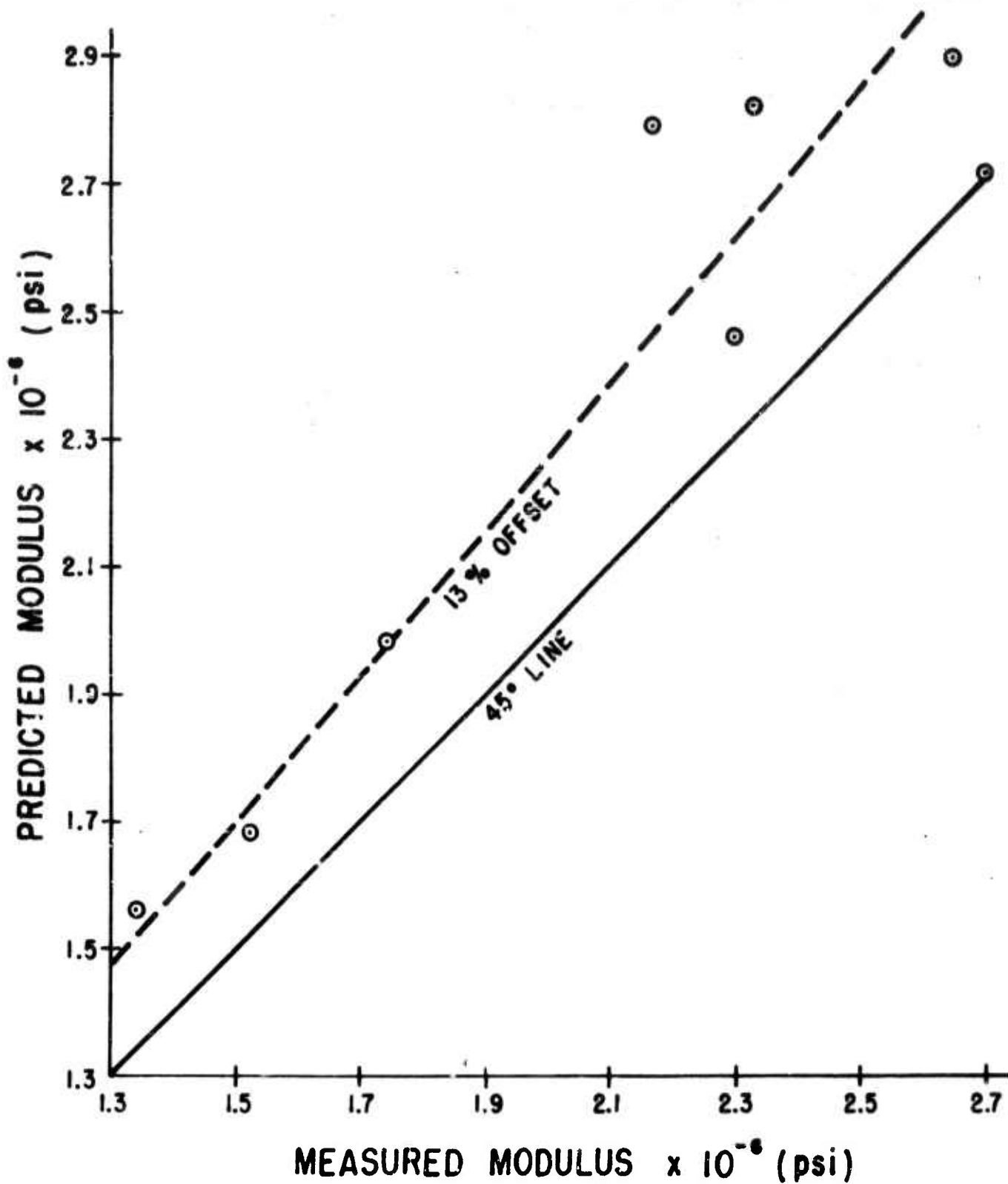Figure 10.   Coordinate System in an Axisymmetric Molding.

Figure 11. A Comparison of Predicted and Measured Young's Moduli in Glass Fiber/Epoxy Transfer Molded Bars.